Fault-Distribution Dependent Reliable Control for T-S Fuzzy Time-Delayed Systems

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The problem of reliable control for T-S fuzzy time-delayed systems is investigated in this paper. A more practical and general actuator-fault-model is proposed by assuming that actuators fault obeys a certain probabilistic distribution. In order to get less conservative results, the state-delay is segmentalized into several continuous equivalent subintervals in constructing the Lyapunov function and stochastic fault information is also introduced in deriving the results. Sufficient conditions for the existence of reliable controller are expressed by a set of linear matrix inequalities. Illustrative examples are exploited to show the effectiveness of the proposed design procedures. [DOI: 10.1115/1.4004066]

Keywords: reliable control, probabilistic failure, delay segmentalized method

1 Introduction

During the past two decades, the study of fault-tolerant control (FTC) has attracted considerable attention because of the growing demands for system reliability in a highly automated industrial system [1–3]. Meanwhile, with the development of fuzzy systems, some reliable fuzzy control design methods have appeared in the field of FTC [1,4–6, and the references therein]. The results cited previously do not involve state time-delays; however, time-delays often occur in many dynamic systems such as biological systems, network systems, and so on. It is shown that the existence of delays usually becomes the source of instability and deteriorating performance of the systems. In recent years, some authors have paid their attention to the problem of reliable control for nonlinear systems with time-delays by using T-S fuzzy models [7–9]. For example, Chen and Liu [7] presented a delay-independent criterion for time-varying delay systems with actuators faults. In literature [8,9], the reliable fuzzy controller design problem of T-S fuzzy descriptor systems and nonuniform sampling system with time-varying delay are proposed, respectively.

Generally, to get less conservative results for time-delayed systems with actuators failures, the researchers mainly focus on to find a better method to handle with time-delay and to establish a reasonable actuator fault model. The existing stability studies for time-delay systems can be classified into two types: delay-independent stability and delay-dependent stability. The delay-independent stability criterion is not affected by the size of the delay; on the other hand, the delay-dependent stability criterion is concerned with the size of the delay and to be less conservative than the delay-independent case [10–12]. As for actuator fault model, usually, it is modeled as $u^F(t) = \Pi u(t)$, where Π is a given scaling factor. In practical systems, the faults, because of actuators aging, zero shift, electromagnetic interference, nonlinear amplification in different frequency field, etc., are varying with circumstance and components themselves in many cases. It will be more reasonable if the fault scale factor obeys a certain probabilistic distribution in an interval. To the best of our knowledge, it seems that there are few results on the actuator failure model satisfying a certain probabilistic distribution. This motivates us to further investigate the problem.

In this paper, first a new actuator failure model that can meet practical situations is proposed. Then, the fuzzy reliable controller with less conservativeness is developed by using the delay-segment method. Finally, two simulation examples are provided to show the effectiveness of the proposed approach.

The paper is organized as follows: The system descriptions and problem formulation are given in Sec. 2. In Sec. 3, a linear matrix inequality (LMI)-based method for the design of reliable fuzzy controllers is presented. Numerical examples are provided to demonstrate the effectiveness of the proposed method in Sec. 4. Finally, the conclusion is drawn in Sec. 5.

2 Problem Formulation

Consider a continuous-time T-S fuzzy system with a constant state delay. The ith rule of the model is described by the following if-then form

$$\begin{aligned} &R^i: \text{If } \theta_1(t) \text{ is } W_1^i \text{ and} \ldots \text{and } \theta_n(t) \text{ is } W_n^i, \text{ then} \\ &\dot{x}(t) = A_i x(t) + A_{di} x(t-\tau) + B_i u(t) \\ &x(t) = \phi(t) \quad t \in [-\tau, 0] \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector and $u(t) \in \mathbb{R}^m$ is the input vector; the initial condition, $\phi(t)$, is a continuous and differential vector valued function of $t \in [-\tau, 0]$; W_i^j is the fuzzy set; $\theta_j(t)(j = 1, 2, \ldots, n)$ is the premise variables; $A_i, A_{di}(i \in \{1, 2, \ldots, r\} \triangleq \mathbb{S})$, and B_i are constant matrices with compatible dimensions.

By using the center-average defuzzifier, product inference, and singleton fuzzifier, the global dynamics of T-S fuzzy system (1) can be expressed as

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} h_i [A_i x(t) + A_{di} x(t-\tau) + B_i u(t)] \\ x(t) = \phi(t) \quad t \in [-\tau, 0] \end{cases} \tag{2}$$

where $h_i = \omega_i(\theta(t)) / \sum_{i=1}^r \omega_i(\theta(t))$, $\omega_i(\theta(t)) = \prod_{j=1}^g W_j^i(\theta_j(t))$, and $W_j^i(\theta_j(t))$ is the membership value of $\theta_j(t)$ in W_j^i, some basic properties of $h(\theta_j(t))$ are $h_i(\theta(t)) \geq 0$, $\sum_{i=1}^r h_i(\theta(t)) = 1$.

Then the following fault model is adopted, with considering the actuators failures, for this study

$$u^F(t) = \sum_{j=1}^{r} \Xi u(t) = \sum_{i=1}^{m} \sum_{j=1}^{r} h_j \xi_i C_i K_j x(t) \tag{3}$$

where, $\Xi = diag\{\xi_1, \ldots, \xi_m\}$ with $\xi_i (i = 1, \ldots, m)$ are m unrelated random variables taking values on interval $[0\ \bar{\xi}]$, where $\bar{\xi} \geq 1$. The mathematical expectation and the variance of ξ_i are μ_i and σ_i^2, respectively, and $C_i = \text{diag}\{\underbrace{0, \ldots, 0}_{i-1}, 1, \underbrace{0, \ldots, 0}_{m-i}\}$. For convenience, we define $\bar{\Xi} = \text{diag}\{\mu_1, \ldots, \mu_m\}$ and $\Delta = \text{diag}\{\sigma_1, \ldots, \sigma_m\}$. $K_j \in \mathbb{R}^{m \times n}$ are feedback gain matrices to be determined.

Remark 1. By introducing a random ξ_i to describe the actuators failure in Eq. (3), it satisfies a certain probabilistic distribution in

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an interval $[0, \bar{\xi}]$ with $\bar{\xi} \geq 1$. For $\xi_i = 0$, it means complete failure of the i th actuator; for $\xi_i = 1$, it means the i th actuator is in good working condition; for $0 < \xi_i < 1$, it means partial failure of the i th actuator; for $\xi_i > 1$, it means the actuator-amplifier with forward drift. It should be noted that, in many cases, the gain of actuators could be larger than normal cases by reasons of the surrounding influence or actuator-amplifiers themselves; therefore, the mathematical expectation μ_i of random variance ξ_i, similar to the scaling factor in Ref. [13], should be defined as $0 < \mu_i < \bar{\mu}_i$, where $\bar{\mu} \geq 1$. Furthermore, σ_i denotes the gain of actuators fluctuation levels because of influence of all the factors acting on actuators.

Combining Eqs. (2) and (3), we obtain the following closed-loop system as follows

$$\dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j [A_{ij}x(t) + B_{ij}x(t) + A_{di}x(t-\tau)] \tag{4}$$

where $A_{ij} = A_i + B_i\bar{\Xi}K_j$ and $B_{ij} = B_i(\Xi - \bar{\Xi})K_j$.

Definition 1. For a given function $V : C^b_{F_0}([-\tau_2, 0], R^n) \times S$, its infinitesimal operator $\mathcal{L}$ [14] is defined as

$$\mathcal{L}V(x_t) = \lim_{\Delta \to 0^+} \frac{1}{\Delta}[\mathcal{E}(V(x_{t+\Delta}|x_t) - V(x_t))] \tag{5}$$

where $\mathcal{E}\{\cdot\}$ stands for the expectation.

3 Main Result

In this section, we aim to develop an innovative approach to guarantee the system (4) is exponentially mean-square stable (EMSS). The controller K_j could be solved from the following results.

THEOREM 1. For given matrices K_j, and scalars τ, d, the system (4) is EMSS if there exists matrices $P > 0, Q_{dn\times dn} > 0, R_i > 0$, and $M_{lij}, N_{lij} (l = 1, \ldots, d+1; i, j \in \mathbb{S})$ such that the following LMIs hold

$$\Pi_{ii} = \begin{bmatrix} \Psi_{ii} & * \\ \Pi^{21}_{ii} & \Pi^{22} \end{bmatrix} < 0 \tag{6}$$

$$\Pi_{ij} = \begin{bmatrix} \Psi_{ij} + \Psi_{ji} & * \\ \bar{\Pi}^{21}_{ij} & \bar{\Pi}^{22} \end{bmatrix} < 0, \quad i < j \in \mathbb{S} \tag{7}$$

where

$$\Psi_{ij} = \begin{bmatrix} \Psi^{11}_{ij} + \Phi + \Phi^T & * & * \\ -M^T & -dR_1 & * \\ -N^T & 0 & -R_2 \end{bmatrix}$$

$$\Psi^{11}_{ij} = \begin{bmatrix} I_{n\times n} \\ 0_{dn\times n} \end{bmatrix}\mathcal{A} + \mathcal{A}^T\begin{bmatrix} I_{n\times n} \\ 0_{dn\times n} \end{bmatrix} + \begin{bmatrix} Q & 0_{dn\times n} \\ 0_{n\times dn} & 0_{n\times n} \end{bmatrix} - \begin{bmatrix} 0_{n\times dn} & 0_{n\times n} \\ I_{dn\times dn} & 0_{dn\times n} \end{bmatrix}\begin{bmatrix} Q & 0_{dn\times n} \\ 0_{n\times dn} & 0_{n\times n} \end{bmatrix}\begin{bmatrix} 0_{n\times dn} & 0_{n\times n} \\ I_{dn\times dn} & 0_{dn\times n} \end{bmatrix}^T$$

$$\mathcal{A} = [PA_{ij} \ \underbrace{0\ldots0}_{d-1} \ PA_{di}]$$

$$\Phi = [M_{ij} + N_{ij} \ -M_{ij} \ \underbrace{0\ldots0}_{d-2} \ -N_{ij}]$$

$$\Pi^{21}_{ii} = [\Lambda^T_{ii} \ \mathcal{C}_{ii}]^T, \Pi^{22} = \mathrm{diag}\{-\mathcal{R} \ -\widetilde{\mathcal{R}}\}$$

$$\bar{\Pi}^{21}_{ij} = [\Lambda^T_{ij} \ \Lambda^T_{ji} \ \mathcal{C}_{ij} \ \mathcal{C}_{ji}]^T$$

$$\bar{\Pi}^{22} = \mathrm{diag}\{-\mathcal{R} \ -\mathcal{R} \ -\widetilde{\mathcal{R}} \ -\widetilde{\mathcal{R}}\}$$

$$M_{ij} = [M_{1ij} \ldots M_{(d+1)ij}],$$

$$N_{ij} = [N_{1ij} \ldots N_{(d+1)ij}]$$

$$\Lambda_{ij} = [\mathcal{R}A_{ij} \ \underbrace{0\ldots0}_{d-1} \ \mathcal{R}A_{di} \ 0 \ 0]$$

$$\mathcal{C}_{ij} = [\mathcal{C}_{1ij}, \ldots, \ \ \mathcal{C}_{lij} \ldots \mathcal{C}_{mij}]$$

$$\mathcal{C}_{lij} = [\sigma_l\mathcal{R}B_iC_lK_j \ \ 0 \ldots 0]^T \ l \in [1, m]$$

$$\mathcal{R} = \frac{\tau^2}{d}R_1 + \tau^2R_2, \widetilde{\mathcal{R}} = \mathrm{diag}\{\underbrace{\mathcal{R}, \ldots, \mathcal{R}}_{m}\}$$

Proof. Construct a Lyapunov–Krasovskii functional candidate as

$$\begin{aligned} V(x_t) = &x^T(t)Px(t) + \int_{t-\frac{\tau}{d}}^{t}\xi^T(s)Q\xi(s)ds \\ &+ \tau\int_{-\frac{\tau}{d}}^{0}\int_{t+s}^{t}\dot{x}^T(v)R_1x(v)dvds \\ &+ \tau\int_{-\tau}^{0}\int_{t+s}^{t}\dot{x}^T(v)R_2\dot{x}(v)dvds \end{aligned} \tag{8}$$

where

$$\xi(t) = \left[x^T(t) \quad x^T(t-\tfrac{\tau}{d}) \ldots x^T(t-\tfrac{(d-1)\tau}{d})\right]^T$$

From the definition of Ξ, we can easily know $\mathcal{E}[B_i(\Xi - \bar{\Xi})K_j] = 0$.

Also, we can have

$$\begin{aligned} &\mathscr{E}\left\{\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r}\sum_{l=1}^{r} h_ih_jB^T_{ij}\mathcal{R}B_{kl}\right\} \\ &\leq \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{m} h_ih_j\sigma_l^2K_j^TC_l^TB_i^T\mathcal{R}B_iC_lK_j \end{aligned}$$

Employing the free-weighting matrix method [15], and the infinitesimal operator (5) for system (4), we have

$$\begin{aligned} &\mathcal{L}V(x_t) \\ &= \mathscr{E}\Bigg\{\sum_{i=1}^{r}\sum_{j=1}^{r}2h_ih_jx^T(t)P[A_{ij}x(t) + A_{di}x(t-\tau)] \\ &+ \xi^T(t)Q\xi(t) - \xi^T(t-\frac{\tau}{d})Q\xi(t-\frac{\tau}{d}) - \tau\int_{t-\frac{\tau}{d}}^{t}\dot{x}^T(s)R_1\dot{x}(s)ds \\ &- \tau\int_{t-\tau}^{t}\dot{x}^T(s)R_2\dot{x}(s)ds + \dot{x}^T(t)\left[\frac{\tau^2}{d}R_1 + \tau^2R_2\right]\dot{x}(t) \\ &+ \sum_{i=1}^{r}\sum_{j=1}^{r}2h_ih_j\zeta^T(t)M_{ij}\left[x(t) - x(t-\tfrac{\tau}{d}) - \textstyle\int_{t-\frac{\tau}{d}}^{t}\dot{x}(s)ds\right] \\ &+ \sum_{i=1}^{r}\sum_{j=1}^{r}2h_ih_j\zeta^T(t)N_{ij}\left[x(t) - x(t-\tau) - \textstyle\int_{t-\tau}^{t}\dot{x}(s)ds\right]\Bigg\} \\ &\leq \mathscr{E}\Bigg\{\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j\Bigg\{x^T(t)\sum_{l=1}^{m}\sigma_l^2K_j^TC_l^TB_i^T\mathcal{R}B_iC_lK_jx(t) \\ &+ x^T(t)A^T_{ij}\mathcal{R}A_{ij}x(t) \\ &+ x^T(t-\tau)A^T_{di}\mathcal{R}A_{di}x(t-\tau) + 2x^T(t)A^T_{ij}\mathcal{R}A_{di}x(t-\tau) \\ &- d\int_{t-\frac{\tau}{d}}^{t}\dot{x}^T(s)dsR_1\int_{t-\frac{\tau}{d}}^{t}\dot{x}(s)ds - \int_{t-\tau}^{t}\dot{x}^T(s)dsR_2\int_{t-\tau}^{t}\dot{x}(s)ds\Bigg\} \end{aligned}$$

where $\zeta(t) = \left[x^T(t) \quad x^T(t-\frac{\tau}{d}) \quad x^T(t-\frac{2\tau}{d}) \quad \cdots \quad x^T(t-\tau)\right]^T$

Hence

$$\begin{aligned} &\mathcal{L}V(x_t) \\ &\leq \mathscr{E}\left\{\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j\eta^T(t)\left\{\Psi_{ij} + \Lambda^T_{ij}\mathcal{R}\Lambda_{ij} + \mathcal{C}_{ij}\widetilde{\mathcal{R}}\mathcal{C}^T_{ij}\right\}\eta(t)\right\} \\ &= \mathscr{E}\Bigg\{\sum_{i=1}^{r}h_i^2\eta^T(t)\left\{\Psi_{ii} + \Lambda^T_{ii}\mathcal{R}\Lambda_{ii} + \mathcal{C}_{ii}\widetilde{\mathcal{R}}\mathcal{C}^T_{ii}\right\}\eta(t) \\ &+ \sum_{i,j=1}^{r}\sum_{i<j}h_ih_j\eta^T(t)\Big\{\Psi_{ij} + \Psi_{ji} + \Lambda^T_{ij}\mathcal{R}\Lambda_{ij} + \Lambda^T_{ji}\mathcal{R}\Lambda_{ji} \\ &+ \mathcal{C}_{ij}\widetilde{\mathcal{R}}\mathcal{C}^T_{ij} + \mathcal{C}_{ji}\widetilde{\mathcal{R}}\mathcal{C}^T_{ji}\Big\}\eta(t)\Bigg\} \end{aligned} \tag{9}$$

where

$$\eta(t) = \left[\zeta^T(t) \ \int_{t-\frac{\tau}{d}}^{t} \dot{x}^T(s)ds \ \int_{t-\tau}^{t} \dot{x}^T(s)ds\right]^T$$

Using Schur complements, it can be shown that Eqs. (6) and (7) are the sufficient conditions for guaranteeing

$$\mathcal{L}V(x_t) < 0 \tag{10}$$

Similar to the method of Ref. [16], we can conclude that the closed-loop systems (4) is EMSS. This completes the proof.

Remark 2. The idea of delay partitioning has appeared in a few literatures, for example, Refs. [10–12 and the references therein], which trigger us for further research on time-delay with new ways and its application to the problem of reliable control. In Eq. (8), d state-vectors are augmented in $\xi(t)$, that is, $[0\,\tau]$ is segmented into d intervals. Although it brings more free matrix, it can lead to significantly less conservative results, which will be illustrated in Sec. 4.

In the following, we are seeking to design the reliable controller gain K_j based on Theorem 1.

THEOREM 2. For given scalars τ_1, τ_2, the system (4) is EMSS if there exist positive definite matrices $X, \tilde{Q}_{dn\times dn}, \tilde{R}_i$, and $\tilde{M}_{lij}, \tilde{N}_{lij}, Y_j (l = 1, \ldots, d+2; i, j \in \mathbb{S})$, such that the following LMIs (11)–(12) hold. Furthermore, the reliable controller gain $K_j = Y_j X^{-1}$.

$$\tilde{\Pi}_{ii} = \begin{bmatrix} \tilde{\Psi}_{ii} & * \\ \tilde{\Pi}_{ii}^{21} & \tilde{\Pi}^{22} \end{bmatrix} < 0 \tag{11}$$

$$\hat{\Pi}_{ij} = \begin{bmatrix} \tilde{\Psi}_{ij} + \Psi_{ij} & * \\ \hat{\Pi}_{ij}^{21} & \hat{\Pi}_{22} \end{bmatrix} < 0, \quad i < j \in \mathbb{S} \tag{12}$$

where

$$\tilde{\Psi}_{ij} = \begin{bmatrix} \tilde{\Psi}_{ij}^{11} + \tilde{\Phi} + \tilde{\Phi}^T & * & * \\ -\tilde{M}^T & -d\tilde{R}_1 & * \\ -\tilde{N}^T & 0 & -\tilde{R}_2 \end{bmatrix}$$

$$\tilde{\Psi}_{ij}^{11} = \begin{bmatrix} I_{n\times n} \\ 0_{dn\times n} \end{bmatrix} \widetilde{\mathcal{A}} + \widetilde{\mathcal{A}}^T \begin{bmatrix} I_{n\times n} \\ 0_{dn\times n} \end{bmatrix} + \begin{bmatrix} \tilde{Q} & 0_{dn\times n} \\ 0_{n\times dn} & 0_{n\times n} \end{bmatrix}$$
$$- \begin{bmatrix} 0_{n\times dn} & 0_{n\times n} \\ I_{dn\times dn} & 0_{dn\times n} \end{bmatrix} \begin{bmatrix} \tilde{Q} & 0_{dn\times n} \\ 0_{n\times dn} & 0_{n\times n} \end{bmatrix} \begin{bmatrix} 0_{n\times dn} & 0_{n\times n} \\ I_{dn\times dn} & 0_{dn\times n} \end{bmatrix}^T$$

$$\widetilde{\mathcal{A}} = [A_i X + B_i \Xi Y_j \ \underbrace{0 \ldots 0}_{d-1} \ A_{di} X]$$

$$\tilde{\Phi} = [\tilde{M}_{ij} + \tilde{N}_{ij} \ \ -\tilde{M}_{ij} \ \underbrace{0 \ldots 0}_{d-2} \ -\tilde{N}_{ij}]$$

$$\tilde{\Pi}_{ii}^{21} = [\tilde{\Lambda}_{ii}^T \ \widetilde{C}_{ii}]^T, \quad \tilde{\Pi}^{22} = \widetilde{R}_{m+1}$$

$$\hat{\Pi}_{ij}^{21} = [\tilde{\Lambda}_{ij}^T \ \tilde{\Lambda}_{ji}^T \ \widetilde{C}_{ij} \ \widetilde{\mathcal{C}}_{ji}]^T, \quad \hat{\Pi}^{22} = \widetilde{R}_{2m+2}$$

$$\tilde{M}_{ij} = [\tilde{M}_{1ij} \ldots \tilde{M}_{(d+1)ij}], \ \tilde{N}_{ij} = [\tilde{N}_{1ij} \ldots \tilde{N}_{(d+1)ij}]$$

$$\tilde{\Lambda}_{ij} = [A_i X + B_i \Xi Y_j \quad 0 \cdots 0 \quad A_{di} X \quad 0 \quad 0]$$

$$\widetilde{\mathcal{C}}_{ij} = [C_{1ij}, \ldots, \quad C_{lij} \ldots C_{mij}], \ \widetilde{\mathcal{C}}_{lij} = [\sigma_l B_i C_l Y_j \quad 0 \ldots 0 \ ]^T$$

$$\bar{R} = \frac{\tau^2}{d} \tilde{R}_1 + \tau^2 \tilde{R}_2$$

$$\widetilde{\mathcal{R}}_k = \mathrm{diag}\{\underbrace{-2\varepsilon X + \varepsilon^2 \bar{R}, \ldots, -2\varepsilon X + \varepsilon^2 \bar{R}}_{k}\}$$

Proof. The proof is cutoff due to space limitation. Contact the authors for the detailed proof.

Remark 3. From Theorem 2, we can see that the criteria depend on not only the state time-delay τ, but also the fault-distribution (μ_i, σ_i) and the number of d.

4 Numerical Examples

In this section, two examples are used to illustrate the merits and effectiveness of the results proposed in this paper. The first example is taken from Ref. [17], from which the advantages of our delay-segment-dependent stability criterion can be shown. The second one, from Ref. [18], is used to show the systems performance when the actuators suffering probabilistic fault by using the proposed reliable controller.

Example 1. Consider T-S fuzzy system (2) with the parameters as following [17]

$$A_1 = \begin{bmatrix} -2 & 0.1 \\ 0 & -0.9 \end{bmatrix}, \ A_2 = \begin{bmatrix} -1 & 0.5 \\ 0 & -1 \end{bmatrix}$$
$$A_{d1} = [-1 \quad 0\backslash 1 \quad -1], \ A_{d2} = \begin{bmatrix} -1 & 0 \\ 0.1 & -1 \end{bmatrix}, B_1 = B_2 = 0$$

Table 1 shows the tendency of the upper bound of τ by using the proposed method closed to the real upper bound with the increasing segment d of τ. Table 2 lists the results the maximum allowable delay bounds derived from various methods including Tian and Peng [19], Chen et al. [20], Peng et al. [21], and the one proposed in this paper. It is seen from Table 2 that the results obtained from our method are less conservative than those obtained from existing methods.

Example 2. Consider the following truck-trailer model with the following parameters [18]

$$A_1 = \begin{bmatrix} 0.5091 & 0 & 0 \\ -0.5091 & 0 & 0 \\ 0.5091 & -4 & 0 \end{bmatrix}, A_2 = \begin{bmatrix} 0.5091 & 0 & 0 \\ -0.5091 & 0 & 0 \\ 0.8102 & -6.3662 & 0 \end{bmatrix}$$
$$A_{d1} = \begin{bmatrix} 0.2182 & 0 & 0 \\ -0.2182 & 0 & 0 \\ 0.2182 & 0 & 0 \end{bmatrix}, A_{d2} = \begin{bmatrix} 0.2182 & 0 & 0 \\ -0.2182 & 0 & 0 \\ 0.3472 & 0 & 0 \end{bmatrix}, \tau = 5$$
$$B_1 = B_2 = \begin{bmatrix} -1.4286 \\ 0 \\ 0 \end{bmatrix}$$

and the fuzzy membership functions are taken as

$$h_1 = \left(1 - \frac{1}{1 + \exp(-3(\theta(t)/0.5 - \pi/2))}\right)$$
$$\left(1 - \frac{1}{1 + \exp(-3(\theta(t)/0.5 + \pi/2))}\right), h_2 = 1 - h_1$$

Supposing the actuator fault-distribution is $\mu_1 = 0.3$, $\sigma_1 = 0.2$, we can get the "reliable controller" from Theorem 2 with $d = 3$, and $\varepsilon = 1$: $K_{r1} = [6.9355 \ -5.7163 \ 0.1170]$, $K_{r2} = [7.2165 \ -7.6963 \ 0.1343]$. Also, we can get the "standard controller", i.e., the systems

Table 1 Upper bound of with increasing (Example 1)

Segment	$d=2$	$d=3$	$d=4$	…
τ	1.904	1.9633	1.9843	…

Table 2 The maximum allowable delay bound without uncertainties (Example 1)

Method	Maximum allowable τ
Tian and Peng [19]	1.5974
Chen et al. [20] (Corollary 1)	1.5974
Peng et al. [21] Corollary 1	1.6341
Corollary 1 (with $d=4$)	1.9843

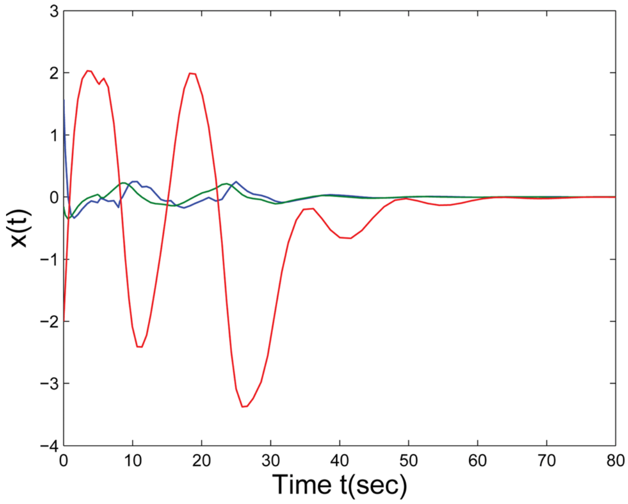

Fig. 1 Responses of $x(t)$ with standard controller K_s

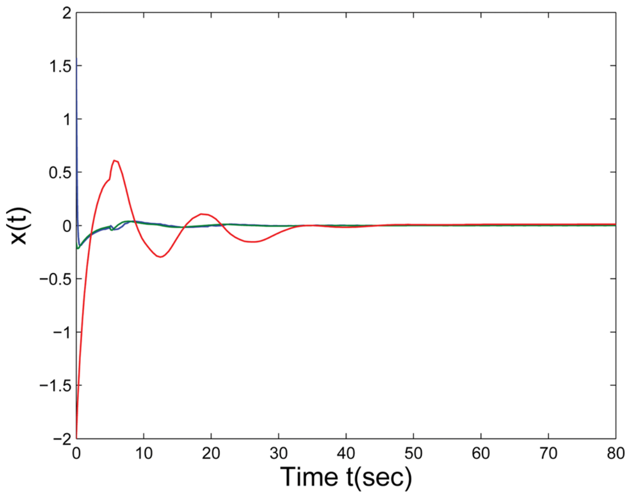

Fig. 2 Responses of $x(t)$ with reliable controller K_r

are in normal case, as $K_{s1} = [2.0807\ \ -1.7149\ 0.0351]$, $K_{s2} = [2.1649\ \ -2.3089\ 0.0403]$.

With the initial condition given by $\phi(t) = [0.5\pi\ \ -0.05\pi\ \ -2]^T$ for $t \in [-5, 0]$. Supposing the actuators fault occurs at [4–25 s]. From Figs. 1 and 2, we can observe that when actuators fault occurs, the closed-loop system with the reliable controller still operates well and maintains an acceptable level of performance. However, the system tends to divergent when using standard controller K_s.

5 Conclusion

In this paper, a new practical actuator fault model is proposed. We concentrate on designing a reliable controller for a class of T-S fuzzy time-delay systems and presenting a less conservative method to achieve closed-loop stability, not only when the system is operating properly but also in the presence of probabilistic actuator failures. Numerical examples are given to illustrate the design procedures.

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